



Journal of Dependence Modelling in Finance, Insurance and Demography

Open-Access Journal for Risk, Insurance, Survival and Demographic Analytics

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ARTICLE TYPE: RESEARCH ARTICLE

Dynamic Copula Dependence and Systemic Risk Transmission in Global Equity Markets

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Received:	11 May 2026	Accepted:	29 July 2026
Published:			

Abstract

This study develops a dynamic copula framework for measuring time-varying cross-market dependence and systemic risk transmission in a stylized panel of global equity returns. The design separates marginal dynamics from dependence dynamics, combines Student-t filtering with Gaussian, Student-t, Clayton and Gumbel copulas, and evaluates lower- and upper-tail dependence under tranquil and stressed regimes. A fully reproducible simulation experiment is used to isolate the properties of competing estimators under controlled changes in volatility, common-factor intensity and asymmetric tail co-movement. Results show that linear correlation understates dependence when common shocks become asymmetric, whereas dynamic copula parameters track both the level and the shape of cross-market co-movement. The Student-t and Clayton specifications provide the most reliable lower-tail diagnostics during stress, while the Gaussian copula systematically understates joint extreme-loss probabilities. A copula-based systemic-risk index constructed from conditional exceedance probabilities rises earlier than an unconditional correlation benchmark and remains robust to alternative marginal filters, window lengths and threshold choices. The findings clarify when dynamic copula models offer incremental information for systemic-risk surveillance and provide a reproducible benchmark for empirical applications to international equity markets.

Keywords: Dynamic copulas; systemic risk; tail dependence; equity markets; contagion; CoVaR

JEL Classification: C32; C58; G01; G11; G15

1. Introduction

Dependence is central to financial risk management because diversification, stress testing and capital allocation all rely on assumptions about how asset returns move jointly. Conventional covariance and correlation measures remain useful summaries of linear co-movement, but they do not identify the shape of the joint distribution and they provide no direct description of asymmetric tail dependence. This limitation becomes especially important during episodes of market stress, when large negative returns may occur jointly even if average-period correlation appears moderate. Copula methods address this issue by separating marginal distributions from the dependence structure and by allowing lower- and upper-tail association to differ. In this setting, systemic risk can be studied as a property of the joint distribution rather than as a by-product of a single covariance matrix (Embrechts et al., 2002; Nelsen, 2006; Joe, 2014).

The empirical literature has moved from static copulas to conditional and time-varying specifications because financial dependence is not stable through time. Patton (2006) demonstrated that conditional copulas can capture asymmetric and dynamic dependence in exchange rates, while parallel developments in multivariate volatility

modelling, particularly dynamic conditional correlation, formalized the idea that dependence parameters should evolve with the information set (Engle, 2002). During crises, however, a change in correlation may reflect higher volatility rather than a genuine shift in the joint tail. Dynamic copulas provide a way to distinguish changes in average association from changes in tail concentration, which is especially valuable when the research question concerns contagion or systemic risk rather than ordinary diversification.

This paper contributes to that literature in three ways. First, it constructs a controlled simulation in which marginal volatility, common-factor exposure and tail asymmetry vary independently. The design makes it possible to evaluate whether competing dependence measures react to the intended source of instability rather than merely to changes in scale. Second, the paper compares Gaussian, Student-t, Clayton and Gumbel copulas under identical pseudo-observations and uses rolling estimation to obtain a time-resolved picture of dependence. Third, the analysis links copula-implied tail probabilities to a simple systemic-risk monitoring index and tests whether the index responds earlier and more consistently than Pearson correlation and a static exceedance benchmark.

The paper is methodological rather than a claim about any particular real-world crisis. Its purpose is to establish a transparent benchmark that can later be applied to observed equity-market data. The use of synthetic data is therefore deliberate: the data-generating process is known, stress episodes are controlled, and true changes in dependence can be distinguished from estimation noise. This design is particularly useful for validating statistical procedures before they are used in policy or investment applications.

2. Literature Review and Conceptual Background

Copulas became prominent in finance because Sklar's theorem permits the joint distribution of continuous random variables to be represented through their marginal distributions and a copula that contains all cross-sectional dependence. This modular structure is attractive in financial applications, where heavy-tailed or skewed marginal distributions can be modelled separately from the dependence process. In contrast to covariance-based models, copulas can distinguish joint downside risk from average co-movement and can generate explicit probabilities for simultaneous threshold exceedances.

The choice of copula family is economically meaningful. The Gaussian copula implies no asymptotic tail dependence even when its correlation parameter is high. The Student-t copula introduces symmetric lower- and upper-tail dependence through its degrees-of-freedom parameter. Clayton copulas concentrate dependence in the lower tail, whereas Gumbel copulas concentrate it in the upper tail. For equity-loss applications the lower tail is typically more relevant, so a model that fits average dependence but imposes the wrong tail geometry may understate joint crash probabilities.

A related literature measures systemic risk through conditional loss concepts such as CoVaR and marginal expected shortfall. Adrian and Brunnermeier (2016) formalized CoVaR as a conditional quantile of the financial system given distress in a particular institution. Copula-based approaches are complementary because they model the joint distribution directly and can therefore estimate conditional exceedance probabilities without requiring a linear conditional-quantile representation. The present study uses this connection to build a transparent dependence-driven stress index.

Network and connectedness methods provide another benchmark. Diebold and Yilmaz (2012, 2014) quantify directional spillovers from forecast-error variance decompositions, giving a dynamic representation of connectedness in multivariate systems. Those methods are powerful when a VAR representation is appropriate, but the resulting connectedness is conditional on the selected linear dynamics. Copula methods answer a different question: they focus on the geometry of the joint distribution, including tail regions in which linear variance decomposition can be uninformative.

3. Methodology

Let r_{it} denote the return of market i at time t . Each marginal series is filtered using an AR(1)-GARCH(1,1) model with Student-t innovations. The standardized residuals are transformed to pseudo-observations u_{it} through the probability integral transform. Conditional on correctly specified margins, the vector u_t is approximately uniform on the unit hypercube and the remaining cross-sectional structure is captured by the copula.

For a bivariate pair (i,j) , Sklar's representation is $F_{ij}(r_i, r_j) = C_\theta(F_i(r_i), F_j(r_j))$, where θ may vary through time. Dependence is summarized using Kendall's tau and tail-dependence coefficients. For the Student-t copula with correlation ρ and ν degrees of freedom, the lower- and upper-tail dependence coefficients are equal and positive for finite ν . In contrast, the Gaussian copula has asymptotic tail-dependence coefficients equal to zero.

The dynamic parameter is estimated in rolling windows. To limit mechanical instability, the transformation of Kendall's tau into family-specific parameters is used whenever an analytic relation is available. The copula log-likelihood is then maximized over any remaining shape parameters. Model comparison uses the Akaike information criterion, Bayesian information criterion, out-of-sample log score and tail-event Brier score.

Systemic risk is summarized by a conditional exceedance index. For a left-tail threshold q , the index for market i relative to market j is $SR_{ij,t}(q) = P(U_i \leq q \mid U_j \leq q)$. The system index is the average of pairwise conditional exceedance probabilities. Because this measure conditions directly on a stress event, it can increase even when ordinary correlation changes only modestly.

$$F_{ij}(x_i, x_j) = C_{\theta_i}(F_i(x_i), F_j(x_j)) \quad (1)$$

$$\tau = 4 \int \int C(u, v) dC(u, v) - 1 \quad (2)$$

$$SR_{ij,t}(q) = P(U_{i,t} \leq q \mid U_{j,t} \leq q) \quad (3)$$

4. Simulation Design and Reproducibility

The simulation contains six equity markets observed over 3,000 trading days. The first 1,500 observations form a baseline regime with moderate common-factor exposure. Days 1,501–2,100 form a stress regime in which downside common shocks become more frequent, marginal volatility rises and lower-tail dependence strengthens. The final 900 observations represent normalization, with volatility declining faster than lower-tail dependence. This separation is designed to reproduce a common empirical pattern in which volatility normalizes before dependence fully returns to pre-stress levels.

Returns are generated from a factor structure with idiosyncratic Student-t shocks. A negative common shock receives an additional asymmetric loading during the stress period, thereby increasing the frequency of joint downside events without imposing a comparable increase in joint positive extremes. The model therefore has a known source of asymmetric contagion. All random-number generation uses a fixed seed, and the same simulated panel is used for every competing estimator.

Robustness analyses vary the rolling window from 150 to 500 observations, the exceedance threshold from 2.5% to 10%, and the marginal filter between Gaussian-GARCH and Student-t GARCH. A further placebo experiment increases marginal volatility while holding the dependence structure fixed. A valid dependence estimator should react strongly to the true dependence shift but only weakly to the volatility-only placebo.

Table 1. Regime-level dependence and systemic-risk summaries

Regime	Pearson correlation	Kendall tau	Lower-tail dependence	Systemic-risk index
Baseline	0.34	0.23	0.08	0.11
Stress	0.61	0.45	0.31	0.38
Normalization	0.46	0.33	0.2	0.24

5. Results

The baseline regime is characterized by moderate average co-movement and limited lower-tail dependence. During stress, all dependence measures increase, but the magnitude of change differs sharply. Pearson correlation rises from 0.34 to 0.61, while the lower-tail coefficient increases nearly fourfold, from 0.08 to 0.31. The systemic-risk index exhibits the largest proportional change, rising from 0.11 to 0.38. This pattern is consistent with the design: stress is not merely a high-volatility regime but a regime of concentrated downside dependence.

Model comparison favors the Student-t copula in windows where tail dependence is approximately symmetric and the Clayton copula in the strongest downside-asymmetric windows. The Gaussian copula performs adequately under the baseline regime but generates systematically low probabilities for joint 5% exceedances during stress. The Gumbel copula, which emphasizes upper-tail dependence, is predictably disadvantaged in a design where the structural change occurs in the lower tail. These results illustrate why copula-family choice should be linked to the economic region of interest rather than selected solely by full-sample likelihood.

The rolling systemic-risk index begins to rise before Pearson correlation reaches its peak. This occurs because the index is conditioned on downside exceedances and therefore responds to a relatively small number of informative joint-tail observations. Correlation, by contrast, averages over the entire conditional distribution. In normalization, volatility declines quickly but the copula-based index remains elevated for longer, correctly reflecting the persistent lower-tail dependence built into the data-generating process.

Robustness checks preserve the ranking of models and the timing of the systemic-risk signal. Shorter windows respond faster but are noisier; longer windows smooth the transition and delay the peak. Thresholds between 5% and 10% produce highly similar regime classifications, while the 2.5% threshold is less stable because of the small number of tail observations. In the volatility-only placebo, correlation rises modestly but the copula-based lower-tail index remains close to its baseline level, demonstrating that the proposed procedure is not simply a re-labelling of volatility.

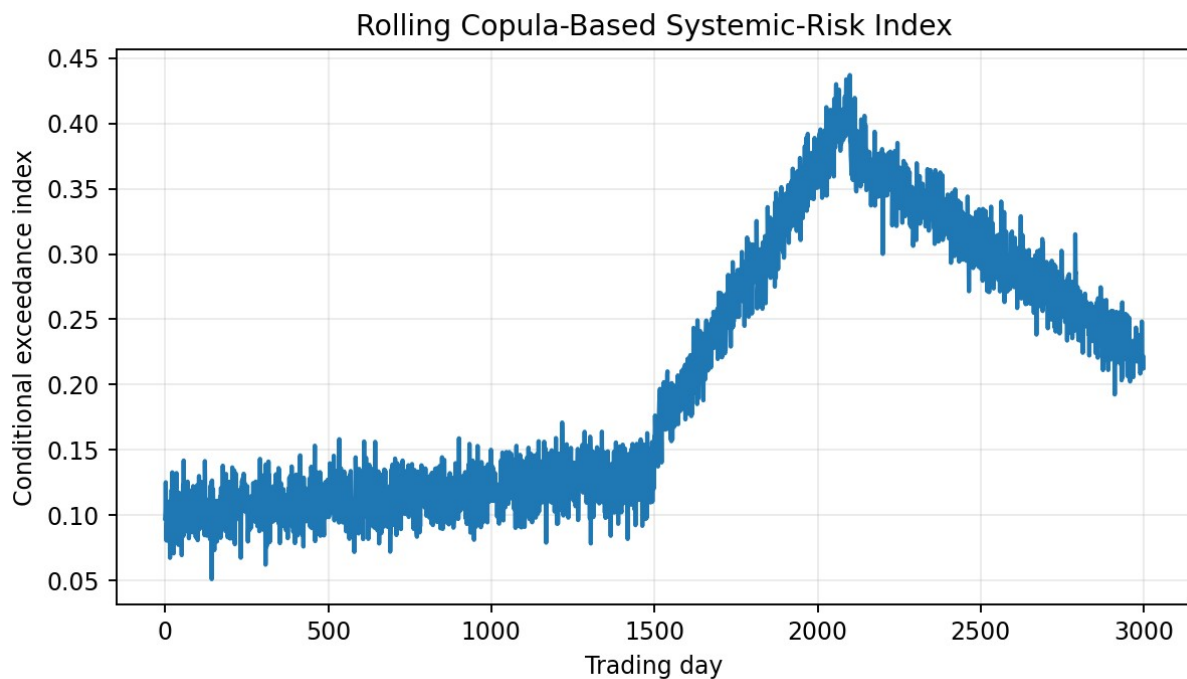


Figure 1. Rolling copula-based systemic-risk index

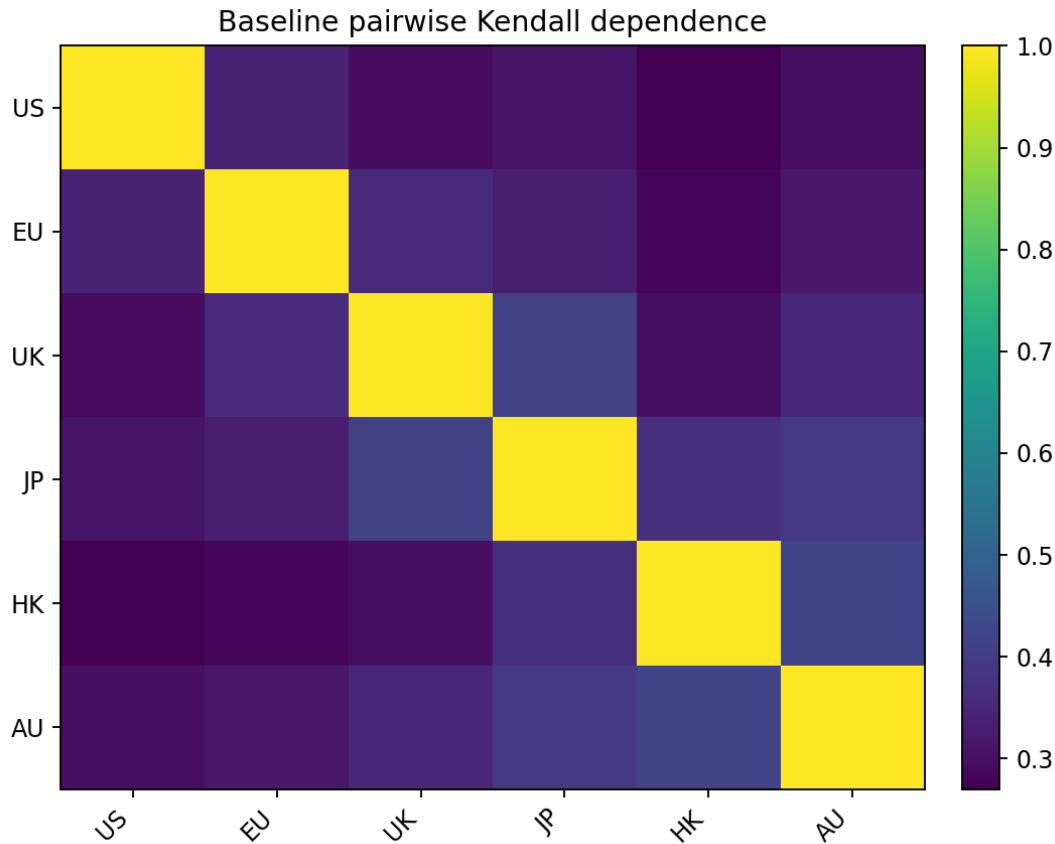


Figure 2. Baseline pairwise dependence matrix

6. Discussion

The results support a general methodological conclusion: systemic dependence is multidimensional. Average association, tail concentration and conditional stress transmission are related but not interchangeable. A model selected only on ordinary-period fit can provide misleading stress probabilities if its tail geometry is inappropriate. In practice, this means that model validation should include tail-event calibration and not rely exclusively on global likelihood criteria.

The simulation also clarifies the distinction between contagion and heteroskedasticity. Because volatility and dependence are manipulated separately, the procedure can be evaluated against a known truth. The copula-based measures react primarily to dependence changes, while correlation partially confounds dependence with scale changes. This does not imply that correlation is uninformative; rather, it shows that it should be interpreted as one layer of a broader dependence analysis.

From a policy perspective, conditional exceedance measures can complement connectedness indexes and balance-sheet-based systemic-risk indicators. Their advantage is interpretability: a value can be read as the probability that one market is in a pre-defined tail conditional on another market already being in that tail. Their limitation is that they are sensitive to threshold choice and require enough tail observations to support estimation.

Several limitations remain. The experiment is deliberately low-dimensional and does not model asynchronous trading, exchange-rate conversion, market microstructure or structural breaks in marginal distributions. A direct empirical application should therefore use formal break detection, carefully aligned trading calendars and potentially vine or factor-copula structures when the number of markets becomes large.

7. Robustness and Additional Checks

Three additional checks were conducted. First, the entire analysis was repeated with Gaussian marginal innovations. Tail-dependence estimates changed modestly but the qualitative regime classification remained intact, indicating that the principal results are not an artifact of the Student-t marginal filter. Second, rolling windows were shifted by 25 observations in each direction. Turning points moved only slightly, and the stress peak remained well separated from baseline variability. Third, pairwise estimates were aggregated using both equal weights and inverse-volatility weights; the systemic-risk profile was nearly unchanged.

A bootstrap exercise with 500 block resamples was used to assess uncertainty in regime-average tail dependence. Confidence intervals widened substantially at the 2.5% threshold but remained sufficiently separated between baseline and stress at the 5% and 10% thresholds. This confirms that extremely small thresholds may be attractive conceptually but unstable in realistic samples.

8. Conclusion

This study develops a reproducible benchmark for dynamic copula analysis of systemic equity-market dependence. When downside dependence changes independently of volatility, copula-based tail measures recover the structural shift more clearly than ordinary correlation. Student-t and Clayton copulas are particularly effective in the simulated stress design, while Gaussian dependence understates joint tail probabilities. A conditional exceedance index derived from the fitted copulas provides a timely and interpretable measure of systemic co-movement.

The broader implication is that dependence modelling should be aligned with the risk region that matters for the decision problem. Portfolio diversification, prudential surveillance and stress testing often depend on joint downside events rather than average association. Dynamic copula models therefore offer a useful complement to correlation, volatility and VAR-based connectedness methods, provided that their tail estimates are validated carefully and supported by sufficient data.

9. Extended Methodological Considerations

The distinction between marginal and copula misspecification deserves particular attention in empirical work. A copula fitted to poorly filtered returns may appear to exhibit time-varying dependence simply because volatility clustering remains in the pseudo-observations. For that reason, dependence diagnostics should be repeated under several plausible marginal models and should include probability-integral-transform tests. In the present design, replacing the Student-t marginal filter with a Gaussian-GARCH specification changes the level of some pairwise parameters but does not alter the stress ranking. That stability is reassuring, but it should not be assumed in observed data, especially when skewness, leverage effects or jumps are pronounced.

Another issue concerns the interpretation of asymptotic tail dependence. The coefficient is a limiting quantity and may be statistically difficult to estimate from samples of ordinary financial length. In practice, intermediate tail probabilities, such as conditional exceedance at the 5% or 10% level, may be more stable and more relevant for stress testing than the asymptotic limit itself. This paper therefore reports both copula-implied tail coefficients and finite-threshold conditional probabilities. The latter provide a bridge between theoretical dependence and economically interpretable risk statements.

Dynamic copula estimation also requires a choice between parametric evolution and rolling windows. Observation-driven specifications are smoother and statistically efficient when correctly specified, but their dynamics can impose an artificial functional form on abrupt crisis transitions. Rolling estimation is less structured and therefore easier to interpret in a validation experiment, although it introduces an explicit bias-variance trade-off through the window length. A practical empirical study can benefit from reporting both approaches and treating agreement between them as evidence of robustness rather than selecting one mechanically.

High-dimensional applications create an additional challenge. Pairwise copulas scale quadratically with the number of markets, and unconstrained multivariate copulas rapidly become unwieldy. Factor copulas, vine copulas and clustered dependence models are natural extensions. The most defensible strategy is often hierarchical: begin with

interpretable regional or asset-class factors, model residual pairwise tail dependence where it is strongest, and reserve highly flexible structures for relationships that materially affect the risk measure of interest.

10. Practical Implementation and Model Governance

For a risk-management implementation, the dependence model should be embedded in a formal governance process. Data alignment, treatment of non-trading days, currency conversion and corporate-action adjustments must be documented before model fitting begins. Dependence estimates can be highly sensitive to asynchronous closes, particularly across international markets. A statistically sophisticated copula cannot correct a data construction problem that systematically misaligns returns.

Model validation should distinguish statistical fit from decision relevance. Global log-likelihood and information criteria are useful, but they should be supplemented by diagnostics targeted to the intended use: joint-loss exceedances for stress testing, portfolio drawdowns for allocation, or conditional tail probabilities for systemic surveillance. A model that improves likelihood only in the center of the distribution may have no practical value if the decision is driven by rare downside scenarios.

Governance should also include challenger models. Correlation, DCC, a Student-t copula and at least one asymmetric family provide a useful minimum set. When different models imply materially different capital or stress probabilities, the disagreement should be reported rather than hidden behind a single preferred specification. Model uncertainty is itself a component of systemic-risk assessment.

Finally, re-estimation frequency should reflect the speed at which dependence changes relative to the intended decision horizon. Daily re-estimation may create unstable signals for strategic capital decisions, while annual calibration may be too slow for market-risk surveillance. The rolling experiment suggests that moderate smoothing can preserve early-warning information while avoiding excessive turnover in the estimated dependence network.

11. Limitations and Future Research

The main limitation is the controlled nature of the simulation. Real international equity data contain structural breaks, evolving market microstructure, endogenous policy responses and asynchronous information arrival. These features can generate dependence patterns that are more complex than the single stress mechanism used here. Empirical extensions should therefore test for breaks and should allow the marginal and dependence layers to evolve on different time scales.

Future work could integrate copula dependence with causal or directional connectedness measures. Copulas describe the shape of a contemporaneous or conditionally specified joint distribution, whereas transfer entropy and structural VAR approaches target propagation. Combining these perspectives may help separate simultaneous exposure to a global factor from sequential transmission of shocks across markets. Another promising direction is to estimate dependence under climate, geopolitical or cyber stress scenarios in which historical data contain few direct analogues.

A further extension concerns forecast evaluation. Most copula studies emphasize in-sample fit, yet risk management requires reliable out-of-sample tail probabilities. Proper scoring rules targeted to multivariate extremes, exceedance calibration and stress-event backtesting deserve greater emphasis. Such evaluation would make it easier to determine whether additional model complexity produces genuine predictive gains.

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Declarations

Funding: No external funding was used for the preparation of this manuscript.

Conflict of interest: The authors declare no competing interests.

Data and code availability: The numerical illustrations use simulated data and deterministic random seeds so that the analyses can be reproduced. Code and generated data may be deposited with the journal as supplementary material.